Published by Institute of Physics Publishing for SISSA/ISAS
Received: *June 12, 2003*
Accepted: *June 18, 2003*

Anomaly, fluxes and (2,0) heterotic-string compactifications

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Abstract: We compute the corrections to heterotic-string backgrounds with (2,0) world-sheet supersymmetry, up to two loops in sigma-model perturbation theory. We investigate the conditions for these backgrounds to preserve spacetime supersymmetry and we find that a sufficient requirement for consistency is the applicability of the $\partial\bar{\partial}$-lemma. In particular, we investigate the α' corrections to (2,0) heterotic-string compactifications and we find that the Calabi-Yau geometry of the internal space is deformed to a hermitean one. We show that at first order in α', the heterotic anomaly-cancellation mechanism does not induce any lifting of moduli. We explicitly compute the corrections to the conifold and to the $U(n)$-invariant Calabi-Yau metric at first order in α'. We also find a generalization of the gauge-field equations, compatible with the Donaldson equations on conformally-balanced hermitean manifolds.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

To the memory of Sonia Stanciu

JHEP06(2003)035

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1. Introduction

The dynamics of the massless superstring modes of the various superstring theories admit a description in terms of ten-dimensional effective supergravities. The latter include, in particular, an infinite tower of α' corrections. From the world-sheet perspective, the equations of the target-space fields are vanishing conditions for the sigma model beta functions, i.e. conditions for conformal invariance. In this approach, α' is the loop-counting parameter in the sigma-model perturbation theory. At zeroth order in α', the vanishing of the beta functions is equivalent to the field equations of ordinary supergravity theories. Terms linear or higher in α' are associated with corrections involving quadratic or higher polynomials in the spacetime curvature and their supersymmetric completions.

For generic heterotic-string backgrounds the α'-corrections to sigma-model couplings are not known. As a result, most configurations relevant to string theory that have appeared in the literature, like the compactifications of [1]–[3], the five-brane [4] and various five-brane intersections [5, 6], are solutions of ordinary supergravity theories. In cases where there is sufficient world-volume supersymmetry, one can argue that higher-order corrections are absent, see [7] for a general argument. In general however, the investigation of stringy effects requires taking the α' corrections into account. One such background is the heterotic five-brane [4] which emerges at one loop in sigma-model perturbation. This and other related results have been extended to three loops in [8].

The low-energy dynamics of the heterotic string is given by $N = 1$ supergravity in ten dimensions. The bosonic fields of the theory are the spacetime metric G, the NS three-form field strength H, the dilaton Φ and the gauge field A. The Green-Schwarz anomaly-cancellation mechanism requires that the three-form Bianchi identity receive an α' correction of the form

$$dH = -\frac{\alpha'}{4}\left(p_1(M) - p_1(E)\right) + O\left(\alpha'^2\right), \tag{1.1}$$

where $p_1(M)$, $p_1(E)$ are the first Pontrjagin forms of spacetime M and of the vector bundle E with connection A, respectively. In the α' expansion for H, $H = T + \alpha' f + O(\alpha'^2)$, the lowest-order term T should be a *closed three-form*, i.e. $dT = 0$. This stems from the fact that at tree-level in the sigma model, the string couples to a two-form gauge potential b, where $T = db$. On the other hand, if $p_1(M) \neq p_1(E)$, then $df \neq 0$ and so $dH \neq 0$. Global anomaly cancellation requires in addition that dH be *exact*, i.e. that H be globally defined.

A class of heterotic-string backgrounds for which the Bianchi identity of the three-form H receives a correction of the type (1.1) are those with (2,0) world-volume supersymmetry. Such models were considered in [9]. The target-space geometry of (2,0)-supersymmetric sigma models has been extensively investigated in [9, 10, 7]. Recently, there is revived interest in these models [11, 12] as string backgrounds and in connection to heterotic-string compactifications with fluxes [13, 14].

In this paper we investigate the α' corrections to heterotic-string backgrounds with (2,0)-world-sheet supersymmetry. We take spacetime to be $M = \mathbb{R}^{10-2n} \times X_n$ and demand that the background preserve 2^{1-n} of spacetime supersymmetry. The $n = 2$ case was examined in [8]. The manifold X_n is hermitean equipped with a compatible connection $\nabla^{(+)}$ with skew-symmetric torsion H, i.e. X_n is Kähler with torsion (KT). At first order in α', the holonomy of the connection $\nabla^{(+)}$ is contained in SU(n) and X_n is conformally balanced, see appendix A.

We show that at linear order in α' such spacetime-supersymmetric backgrounds satisfy the anomaly-cancellation condition and the field equations. In the proof, we make use of the results of [16] summarised in appendix A of the present paper. We find that the

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corrections to the other fields are determined by the corrections to the metric. We stress that *consistency* of the anomaly cancellation condition with the field equations requires that in the latter we include the *two-loop* contribution. A sufficient condition for the consistency of spacetime supersymmetry with the anomaly cancellation and the field equations, is the applicability of the $\partial\bar{\partial}$-lemma[1] on X_n.

We also consider the Donaldson equations on a non-Kähler hermitean manifold. These are related to the gaugino Killing-spinor equation. For generic hermitean manifolds, it is not apparent that the Donaldson equations are associated with a second-order equation for the gauge connection, i.e. a field equation. We find that they are, however, if the underlying hermitean manifold is conformally balanced.

It has been shown in [15] that if X_n is compact, nonsingular and all fields are smooth, then T vanishes and to zeroth order in the α' expansion X_n is a Calabi-Yau n-fold. We shall take this to be the starting point of the α' expansion. Proceeding to first order in α', X_n is deformed to a conformally-balanced KT manifold with $\mathrm{hol}(\nabla^{(+)}) \subseteq \mathrm{SU}(n)$. We determine the deformations of the fields using Hodge theory. Moreover, we compute the dimension of the moduli space and we find it to be the same as that of the moduli space of the underlying Calabi-Yau manifold. We therefore conclude that at this order in α', there is *no lifting* of moduli in (2,0) compactifications with $T = 0$. As particular examples of the general theory mentioned above, we compute the $\mathcal{O}(\alpha')$ corrections to the conifold [17] and to the $U(n)$-invariant Calabi-Yau metric found in [18]. We find that the singularity of the conifold persists to first order in α'. For other α' corrections to conifold geometry see [20].

This paper is organised as follows: In section 2, we establish our notation and write down the field and the Killing-spinor equations for the heterotic string, up to two loops in sigma-model perturbation theory (order $\mathcal{O}(\alpha')$). In section 3, we give the conditions on the deformations of the metric required by spacetime supersymmetry, and express the deformations of the NS three-form and the dilaton in terms of those of the metric. We then show that the Killing-spinor equations for the aforementioned set of fields imply the field equations at this order in α', provided the anomalous Bianchi identity of the H field is satisfied. In section 4, we show that the gaugino Killing-spinor equation, which is equivalent to the Donaldson equations on a hermitean manifold, implies the field equation for the gauge connection. In section 5, we compute the $\mathcal{O}(\alpha')$ corrections to the fields and show that the dimension of the moduli space is the same as that of the moduli space of the Calabi-Yau space we started with at zeroth order in α'. In section 6 and seven, we compute the $\mathcal{O}(\alpha')$ corrections to the conifold and to the Calabi metric, respectively. In section 8, we discuss the consequences of our results in the context of compactifications with fluxes and we comment on α' corrections beyond two-loops. In appendix A, we summarise some of the properties of KT geometry. In appendix B, we explain the relation between the Lichnerowicz and Laplace operators. Finally, in appendix C we give a solution to the field equations at linear order in α' without the use of the Killing-spinor equations.

[1]The $\partial\bar{\partial}$-lemma is not valid on all non-Kähler hermitean manifolds.

2. Field and Killing-spinor equations

The bosonic fields of the ten-dimensional supergravity which arises as low energy effective theory of the heterotic string are the spacetime metric G, the NS three-form field strength H, the dilaton Φ and the gauge connection A. We define the connections

$$\nabla^{(\pm)}_M Y^N = \nabla_M Y^N \pm \frac{1}{2} H^N{}_{MR} Y^R ,$$

where ∇ is the Levi-Civita connection of the metric G and $M, N, R = 0, 1 \ldots, 9$ are spacetime indices. The three form H has an expansion in α' of the form[2]

$$H = T - \frac{\alpha'}{4} \left(Q_3 \left(\Gamma^{(-)} \right) - Q_3(A) \right) + O(\alpha'^2) , \tag{2.1}$$

where T is a closed three-form, $dT = 0$ and Q_3 are Chern-Simons three-forms. We have

$$dH \equiv -\alpha' P + O(\alpha'^2) , \qquad P = \frac{1}{4} \left[\mathrm{tr} \left(R^{(-)} \wedge R^{(-)} \right) - \mathrm{tr}(F \wedge F) \right] , \tag{2.2}$$

where the trace on the gauge indices is taken as

$$\mathrm{tr}\, F \wedge F = F^a{}_b \wedge F^b{}_a , \qquad F = dA + A^2 .$$

Similarly for the trace of $R^{(-)}$, where $R^{(-)}$ is the curvature of the connection $\nabla^{(-)}$. The four-form P is proportional to the difference of the Pontrjagin forms of the tangent bundle of spacetime and Yang-Mills bundle of the heterotic string.

The string frame field equations of the heterotic string up to two-loops [19] in sigma model perturbation theory are

$$\begin{aligned} R_{MN} + \frac{1}{4} H^R{}_{ML} H^L{}_{NR} + 2 \nabla_M \partial_N \Phi + & \\ + \frac{\alpha'}{4} \left[R^{(-)}{}_{MPQR} R^{(-)}{}_N{}^{PQR} - F_{MPab} F_N{}^{Pab} \right] + O(\alpha'^2) &= 0 \\ \nabla_M (e^{-2\Phi} H^M{}_{RL}) + O(\alpha'^2) &= 0 \\ \nabla^{(+)M} (e^{-2\Phi} F_{MN}) + O(\alpha'^2) &= 0 , \end{aligned} \tag{2.3}$$

where we have suppressed the gauge indices. The field equation of the dilaton Φ is implied from the first two equations above. Our curvature conventions are given in appendix A.

Let $\{\Gamma^M; M = 0, \ldots, 9\}$ be a basis of the Clifford algebra $\mathrm{Cliff}(\mathbb{R}^{1,9})$, i.e. $\Gamma^M \Gamma^N + \Gamma^N \Gamma^M = 2 G^{MN}$. Then the string frame Killing-spinor equations[3] are [10, 21]

$$\begin{aligned} \nabla^{(+)} \epsilon + O(\alpha'^2) &= 0 \\ \left(\Gamma^M \partial_M \Phi - \frac{1}{12} H_{MNR} \Gamma^{MNR} \right) \epsilon + O(\alpha'^2) &= 0 \\ F_{MN} \Gamma^{MN} \epsilon + O(\alpha'^2) &= 0 , \end{aligned} \tag{2.4}$$

[2] Our form conventions are $\omega_k = \frac{1}{k!} \omega_{i_1, \ldots, i_k} dx^{i_1} \wedge \cdots \wedge dx^{i_k}$.

[3] We have used the notation $\Gamma^{M_1 \ldots M_k} = \Gamma^{[M_1} \ldots \Gamma^{M_k]}$.

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where ϵ is a section of the spin bundle S_+.[4] It is clear that the first Killing spinor equation is a parallel transport equation for the connection $\nabla^{(+)}$. Since the connection of the spin bundle S_+ is induced from the tangent bundle of spacetime, the investigation of this Killing-spinor equation is greatly simplified. The first, second and third Killing-spinor equations are associated with the supersymmetry transformations of the gravitino, dilatino and gaugino, respectively. We shall use this terminology in what follows to distinguish between them.

It is clear from the field equations that the two-loop contribution to the Einstein equations is at the same order as the modification of the torsion H due to the cancellation of the heterotic anomaly. Consistency then requires that both should be taken into account. The various field and Killing-spinor equations are expected to receive corrections to all orders in α'. Therefore a solution of the field and/or Killing-spinor equations of the effective supergravity theory can be expanded as

$$\begin{aligned} G &= g + \alpha' h + O(\alpha'^2) \\ H &= T + \alpha' f + O(\alpha'^2) \\ \Phi &= \varphi + \alpha' \phi + O(\alpha'^2) \\ A &= B + \alpha' Q + O(\alpha'^2) \,. \end{aligned} \tag{2.5}$$

In this expansion, the fields (g, T, φ, B) solve the field and the Killing-spinor equations at zeroth-order in α'. We again remark that $dT = 0$ although dH may *not* vanish, $dH \neq 0$. The deformation (h, f, ϕ, Q) linear in α' is the first-order correction to the background (g, T, φ, B). Of course, the fields receive higher-order corrections in α'. In what follows, we determine the deformations (h, f, ϕ, Q) by requiring that (G, H, Φ, A) in (2.5) solve the field (2.3) and Killing-spinor (2.4) equations.

3. The α' corrections to backgrounds with torsion

3.1 World-sheet and spacetime supersymmetry

We restrict our attention to heterotic string backgrounds of the form

$$\begin{aligned} ds^2 &= ds^2(\mathbb{R}^{10-2n}) + ds^2(X_n) \\ T &= \frac{1}{3!} T_{ijk}(y) dy^i \wedge dy^j \wedge dy^k \\ \varphi &= \varphi(y) \\ B &= B_i(y) dy^i \,, \end{aligned} \tag{3.1}$$

where $\{y^i; i = 1, \dots, 2n\}$ are coordinates on a manifold X_n, $n \leq 4$, and $dT = 0$ as we have explained in the introduction.

In addition, we require that the background (g, T, φ, B) be compatible with (2,0) world-sheet supersymmetry. This means that the light-cone gauged fixed string world-sheet

[4] The spin group $\mathrm{Spin}(1,9)$ has two inequivalent irreducible sixteen-dimensional spinor representations and $S_\pm$ are the associated bundles.

action is (2,0)-supersymmetric. In particular this implies that X_n is a hermitean manifold, (X_n, J, g), with complex structure J which is parallel with respect to $\nabla^{(+)}$ connection, i.e. (X_n, J, g) is a KT manifold. The torsion T of KT manifolds is specified by the metric g and the complex structure J, see appendix A.

The background (3.1) is expected to receive α' corrections because the supergravity field equations are modified by two- and higher-loop contributions in sigma model perturbation theory and in particular by the heterotic anomaly-cancellation mechanism. After these corrections are included, the background is expected to be of the form

$$\begin{aligned} ds^2 &= ds^2(\mathbb{R}^{10-n}) + d\tilde{s}^2(X_n) \\ H &= \frac{1}{3!} H_{ijk}(y) dy^i \wedge dy^j \wedge dy^k \\ \Phi &= \Phi(y) \\ A &= A_i(y) dy^i \,, \end{aligned} \tag{3.2}$$

where $d\tilde{s}^2(X_n) = G_{ij}(y) dy^i dy^j$. The three-form H is *not* necessarily closed, because of (2.2).

As we have seen, sigma model loop effects and the heterotic anomaly cancellation mechanism alter the geometry of the manifold X_n. Nevertheless, it is expected that if the original manifold (X_n, J, g) has a KT structure, the geometry, after the corrections are taken into account, remains KT. So the manifold (X_n, J, G) has a KT structure as well but now the torsion H is *not* closed. This is because it is expected that there is a scheme which preserves the (2,0) world-volume supersymmetry in sigma model perturbation theory [22].

A solution (g, T, φ, B) of the zeroth order in α' field equations associated with KT manifold (X_n, J, g) does *not* necessarily satisfy the Killing-spinor equations (2.4) of supergravity theory. The conditions for (g, T, φ, B) to satisfy the gravitino, dilatino and gaugino Killing-spinor equations [10, 15] are

$$\begin{aligned} \mathrm{hol}(\nabla^{(+)}) &\subseteq \mathrm{SU}(n) \,, & \theta &= 2d\varphi \\ F(B)_{2,0} &= F(B)_{0,2} = 0 \,, & \Omega^{ij} F(B)_{ij} &= 0 \,, \end{aligned} \tag{3.3}$$

where $\mathrm{hol}(\nabla^{(+)})$ is the holonomy of the connection $\nabla^{(+)}$, $\Omega_{ij} = g_{jk} J^k{}_j$ is the Kähler form and θ is the Lee form of the hermitean geometry. (The Lee-form has been given in appendix A). KT manifolds for which the Lee-form is *exact* are called *conformally balanced.* The conditions on the curvature $F(B)$ of the gauge connection B required by the gaugino Killing-spinor equations imply that F is a (1,1)-form with respect to the complex structure J and its trace with Ω vanishes. I.e. considered as a two-form $F(B)$ takes values in the Lie algebra of $\mathrm{SU}(n)$. These conditions are the analogue of the Donaldson equations for hermitean manifolds. It can be shown that the backgrounds of (3.3) preserve 2^{1-n} of spacetime supersymmetry.

Conversely if (g, T, φ, B) in (3.1) satisfies the Killing-spinor equations (2.4) preserving 2^{1-n} of spacetime supersymmetry, then X_n is a conformally balanced KT manifold and the holonomy of $\nabla^{(+)}$ is contained in $\mathrm{SU}(n)$.

As we have explained, the geometry of the background (G, H, Φ, A) (3.2) is expected to be KT. However, it is not apparent that if the (g, T, φ, B) background is spacetime supersymmetric, then (G, H, Φ, A) will also be spacetime supersymmetric. The corrections to Killing-spinor equations of supergravity (3.3) at order $\mathcal{O}(\alpha')$ are determined by the corrections to the metric and the torsion but otherwise their dependence on the metric and the torsion remains the same.[5] This has the consequence that if we insist that the corrected background (G, H, Φ, A) preserve the same number of supersymmetries as (g, T, φ, B), then (X_n, J, G) is again a KT manifold for which $\mathrm{hol}\nabla^{(+)} \subseteq \mathrm{SU}(n)$, $\theta = 2d\Phi$ and $F(A)_{2,0} = F(A)_{0,2} = 0$, $\Omega^{ij} F_{ij} = 0$. In this case, $\nabla^{(+)}, \theta$ and Ω are the connection, Lee form and Kähler form of the metric G, respectively.

We conclude that at linear order in α', the corrections to the geometry of X_n are deformations which preserve the following two properties:

- X_n is a conformally balanced KT manifold and
- the holonomy $\nabla^{(+)}$ is contained in $\mathrm{SU}(n)$.

In what follows, we derive the conditions on the deformations of the geometry which preserve the above properties and we solve the field and Killing-spinor equations to first order in α'. We also present a similar analysis for the conditions on the gauge connection.

3.2 Gravitino and dilatino Killing-spinor equations

As we have mentioned, in order to solve the gravitino and dilatino Killing-spinor equations, we have to specify the deformations $(G, H) = (g + \alpha' h, T + \alpha' f)$ which preserve the properties that (X_n, J, g) is conformally balanced KT manifold and $\mathrm{hol}(\nabla^{(+)}) \subseteq \mathrm{SU}(n)$.

First, we consider deformations which preserve the hermiticity of the metric with respect to the complex structure J. This means that $h_{\alpha\beta} = 0$, where $\alpha, \beta = 1, \ldots, n$ are labels for the holomorphic coordinates on X_n. It can then be easily shown that $\mathrm{hol}(\nabla^{(+)}) \subseteq U(n)$ provided that the deformation for the torsion is

$$f_{\alpha\beta\bar{\gamma}} = -\nabla_\alpha h_{\beta\bar{\gamma}} + \nabla_\beta h_{\alpha\bar{\gamma}}\,, \tag{3.4}$$

where ∇ is the Levi-Civita connection of the metric g, $f_{\bar{\alpha}\bar{\beta}\gamma} = (f_{\alpha\beta\bar{\gamma}})^*$ and the rest of the components vanish. The latter is required because the torsion of a KT geometry is a (2,1)- and (1,2)-form.

Furthermore, the deformation of the connection of the canonical bundle[6] of X_n associated with $\nabla^{(+)}$ is

$$\omega_\alpha(G) = \omega(g)_\alpha + \alpha' \left[2i\nabla_\beta h_\alpha{}^\beta - i\nabla_\alpha h^\beta{}_\beta + iT_{\bar{\delta}\beta\bar{\gamma}} g^{\beta\bar{\gamma}} h_{\alpha\bar{\delta}} - iT_{\alpha\beta\bar{\gamma}} h^{\beta\bar{\gamma}}\right] + O(\alpha'^2)\,, \tag{3.5}$$

where $\omega(g)$ is the connection of the canonical bundle associated with the connection[7] $\nabla^{(+)}(g)$ and $\omega_{\bar{\alpha}} = (\omega_\alpha)^*$. A necessary and sufficient condition for $\mathrm{hol}(\nabla^{(+)}) \subseteq \mathrm{SU}(n)$ is

[5] It is not expected that this property of the Killing spinor equations persists to all orders in sigma model perturbation theory. The dependence of the Killing-spinor equations on the metric and the torsion will change at higher orders [23].

[6] This is the diagonal U(1) part of a $U(n)$ connection on the tangent bundle.

[7] To avoid confusion, sometimes we use the notation $\nabla^{(+)}(g)$ to denote the connection $\nabla^{(+)}$ with respect to the metric g.

that the curvature of the canonical bundle vanishes. For the connection (3.5), a sufficient condition is

$$2i\nabla_{\beta}h_{\alpha}{}^{\beta} - i\nabla_{\alpha}h^{\beta}{}_{\beta} + iT_{\bar{\delta}\beta\bar{\gamma}}g^{\beta\bar{\gamma}}h_{\alpha\bar{\delta}} - iT_{\alpha\beta\bar{\gamma}}h^{\beta\bar{\gamma}} = 0\,, \tag{3.6}$$

where $\nabla = \nabla(g)$.

It remains to find the condition required for X_n to remain conformally balanced after the deformation. For this, we compute the first-order deformation of the Lee form to find

$$\theta_{\alpha} = \theta(g)_{\alpha} + \alpha'\left[-\nabla_{\alpha}(g^{\beta\bar{\gamma}}h_{\beta\bar{\gamma}}) - \nabla_{\beta}h_{\alpha\bar{\gamma}}g^{\beta\bar{\gamma}} + \frac{1}{2}T_{\alpha\beta\bar{\gamma}}h^{\beta\bar{\gamma}} - \frac{1}{2}H^{\bar{\delta}}{}_{\beta\bar{\gamma}}g^{\beta\bar{\gamma}}h_{\alpha\bar{\delta}}\right], \tag{3.7}$$

where $\theta(g)$ is the Lee-form of the (g, J) geometry. Substituting (3.6) in (3.7), we find that

$$\theta_{\alpha} = \theta(g)_{\alpha} + \frac{\alpha'}{2}\nabla_{\alpha}(g^{\beta\bar{\gamma}}h_{\beta\bar{\gamma}})\,. \tag{3.8}$$

The dilatino Killing-spinor equation can be solved by setting

$$\phi = \frac{1}{4}g^{\beta\bar{\gamma}}h_{\beta\bar{\gamma}}\,. \tag{3.9}$$

Therefore the dilaton is deformed to

$$\Phi = \varphi + \frac{\alpha'}{4}g^{\beta\bar{\gamma}}h_{\beta\bar{\gamma}} + O(\alpha'^2)\,.$$

The equations that remain to be solved are those in (3.6). Observe that these equations are $2n$ in number, i.e. as many as the diffeomorphisms of X_n. Since there is some redundancy in specifying the deformation h up to an infinitesimal diffeomorphism generated by the vector field v, i.e. $h'_{\alpha\bar{\beta}} = h_{\alpha\bar{\beta}} + \nabla_{\alpha}v_{\bar{\beta}} + \nabla_{\bar{\beta}}v_{\alpha}$, it is expected on physical grounds that it is always possible to choose an h so that (3.6) is satisfied. Therefore (3.6) can be viewed as a choice of gauge fixing for diffeomorphisms of X_n. In the following, we provide more evidence that this is a good gauge choice.

It remains to examine the conditions on the gauge connection. We postpone this for after the investigation of the field equations of the metric and the two-form gauge potential.

3.3 The solutions of field equations

Having derived the conditions for the deformations to satisfy the gravitino and dilatino Killing-spinor equations, we now focus on the solutions of the field equations for the metric and the NS two-form potential. In particular we show that at order α' both these field equations are satisfied provided that the heterotic anomaly-cancellation condition holds.

Assuming that the background (g, T, φ) satisfies the field equations at zeroth order in α', substituting (2.5) in the field equation for the metric (2.3) and collecting the terms linear in α', we find

$$\begin{aligned}\Delta_L h_{ij} - \frac{1}{4}T_{imn}f_j{}^{mn} - \frac{1}{4}T_{jmn}f_i{}^{mn} + \frac{1}{2}h^{mn}g^{kl}T_{imk}T_{jnl} + \\ +2\nabla_i\partial_j\phi - g^{kl}\left(\nabla_i h_{jk} + \nabla_j h_{ik} - \nabla_k h_{ij}\right)\partial_l\phi_0 + S_{ij} = 0\,,\end{aligned} \tag{3.10}$$

where Δ_L is the Lichnerowicz operator with respect to the metric g (see appendix B) and

$$S_{ij} = \frac{1}{4}\left[R^{(-)}{}_{iklm}R^{(-)}{}_j{}^{klm} - F_{ikab}F_j{}^{kab}\right]$$

is the two loop contribution to the beta function. The curvature $R^{(-)}$ is with respect to (g, T) and $F = F(B)$.

Clearly, (3.10) is rather involved. To proceed, we take the original background (g, T, φ, B) to be spacetime supersymmetric in the sense described in the previous section. In addition we assume that after the deformation the background remains supersymmetric. As we have seen this is equivalent to requiring that the KT geometry (X_n, J, G) be conformally balanced and $\mathrm{hol}(\nabla(G)^{(+)}) \subseteq \mathrm{SU}(n)$. Substituting the deformation (2.5) in (A.4) of appendix A and collecting the terms linear in α', we find that

$$\begin{aligned}\Delta_L h_{ij} - \frac{1}{4}T_{imn}f_j{}^{mn} - \frac{1}{4}T_{jmn}f_i{}^{mn} + \frac{1}{2}h^{mn}g^{kl}T_{imk}T_{jnl} + \\ +2\nabla_i\partial_j\phi - g^{kl}(\nabla_i h_{jk} + \nabla_j h_{ik} - \nabla_k h_{ij})\partial_l\phi_0 = \frac{1}{4}J^k{}_i df_{kjmn}\Omega^{mn}\,.\end{aligned} \quad (3.11)$$

Observe that there is no explicit contribution of the metric deformation on the right-hand-side of (A.4). This is because at zeroth order the torsion is closed, $dT = 0$. Using (3.10) and (3.11), we find that

$$\frac{1}{4}J^k{}_i df_{kjmn}\Omega^{mn} + S_{ij} = 0\,. \quad (3.12)$$

Next consider the anomaly-cancellation condition (2.2) which to linear order in α' can be written as

$$df = -P\,, \quad (3.13)$$

where P depends on (g, T, B). Since we have assumed that the background (g, T, φ, B) is supersymmetric, $R^{(-)}$ and F satisfy the conditions

$$\begin{aligned} R^{(-)i}_{mn}{}_j J^m{}_k J^n{}_l &= R^{(-)i}_{kl}{}_j\,, \qquad & \Omega^{mn}R^{(-)i}_{mn}{}_j &= 0 \\ F_{mn}{}^a{}_b J^m{}_k J^n{}_l &= F_{kl}{}^a{}_b\,, \qquad & \Omega^{mn}F_{mn}{}^a{}_b &= 0\,. \end{aligned} \quad (3.14)$$

These conditions on $R^{(-)}$ can be easily deduced from the fact that the holonomy of $\nabla^{(+)}$ is contained in $\mathrm{SU}(n)$ and $R^{(-)}{}_{ij,kl} = R^{(+)}{}_{kl,ij}$ provided that the torsion is closed ($dT = 0$). The conditions on F can be deduced from the Killing-spinor equations of the gaugino. Contracting the anomaly-cancellation condition[8] (3.13) with the Kähler form Ω and using (3.14), it is easy to see that the field equation for the metric (3.12) is satisfied.

In order to solve the field equations for the metric, it is sufficient to solve the anomaly-cancellation condition (3.13). Substituting (3.4) in (3.13), we find that

$$P = -2i\partial\bar{\partial}Y\,, \quad (3.15)$$

where $Y_{ij} = h_{ik}J^k{}_j$. The global anomaly-cancellation condition requires that P be exact. Since P is an exact, real (2,2)-form, if the $\partial\bar{\partial}$-lemma is valid on X_n, then there is a real

[8]This derivation of (3.12) from (3.13) is sensitive to the relative numerical coefficient of the two-loop contribution to the metric field equation and that of the anomaly-cancellation condition.

(1,1)-form Y *globally defined* on X_n such that (3.15) is satisfied. On hermitean manifolds which are *not* Kähler, the $\partial\bar{\partial}$-lemma is *not* valid in general. Therefore a *sufficient* condition for the existence of spacetime supersymmetric solutions in backgrounds with non-vanishing torsion, $T \neq 0$, is the *validity* of the $\partial\bar{\partial}$-lemma.

The solution to the anomaly-cancellation condition (3.15) is not unique. Indeed, if Y satisfies (3.15), then

$$Y' = Y + \partial\bar{w} + \bar{\partial}w\,, \tag{3.16}$$

where w is a (1,0)-form, is also a solution. This gauge freedom in Y is equivalent to specifying the deformation h in the metric up to an infinitesimal diffeomorphism. This can be easily seen by setting $v = -iw$. Therefore having determined Y from (3.15), we still have the gauge freedom to solve the supersymmetry condition (3.6).

The solution of (3.15) up to a gauge transformation (3.16) is not unique. The classes of independent solutions are described by the Aeppli group $V^{1,1}(X_n)$ defined by

$$V^{1,1} = \frac{\mathrm{Ker}(i\partial\bar{\partial} : \Lambda^{1,1}(X_n) \to \Lambda^{2,2}(X_n))}{\partial\Lambda^{0,1}(X_n) + \bar{\partial}\Lambda^{1,0}(X_n)}\,,$$

see [24] for a related discussion. The dimension of this group is the dimension of the moduli space of solutions of (3.15). However it is not apparent that all elements of this group are associated with spacetime-supersymmetric deformations. The latter should in addition satisfy (3.6).

It remains to show that the field equations of the NS two-form gauge potential are satisfied as well. The proof of this is based on an identity shown in [16, (corollary 3.2)] which can be stated as follows: Let (X_n, J, G) be a conformally balanced KT manifold with torsion H, $dH \neq 0$, and $\mathrm{hol}(\nabla^{(+)}) \subseteq \mathrm{SU}(n)$, then

$$\nabla^i H_{ijk} = \theta_i\, H^i{}_{jk}\,. \tag{3.17}$$

This statement is valid irrespectively of whether or not G is a small perturbation of another metric g.

Both KT structures (X_n, J, g) and (X_n, G, J) satisfy the aforementioned conditions because they are supersymmetric. Therefore both satisfy (3.17) with their respective torsions and Lee forms. Using (3.17) for the α' corrected background (G, H, Φ, A), the field equation (2.3) for the NS two-form

$$-2\partial_i\Phi H^i{}_{jk} + \nabla^i H_{ijk} + O(\alpha'^2) = 0 \tag{3.18}$$

can be written as

$$(\theta_i - 2\partial_i\Phi)H^i{}_{jk} + O(\alpha'^2) = 0$$

which vanishes identically because (X_n, J, G) is conformally balanced. Since the field equations for the background (g, T, φ, B) are satisfied by assumption and as we have shown the field equations (3.18) for (G, H, Φ, A) are satisfied as well, the part of (3.18) linear in α' vanishes identically. Therefore the field equations for the NS two-form gauge potential are satisfied without additional conditions on the deformation h of the metric.

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4. The gauge field

The main purpose of this section is to show that the Killing-spinor equations of the gaugino imply the field equations of the gauge field before and after the α' corrections are taken into account. The simplest way to show this is by investigating the properties of gauge fields on conformally balanced KT manifolds.

4.1 Gauge fields on KT conformally balanced manifolds

We first describe the well-known relation between the Donaldson equations and the field equations of a gauge connection on a Kähler manifold. Let E be a vector bundle over a *Kähler* manifold (M, J, G) equipped with a connection A with curvature F. If A satisfies the Donaldson equations

$$F_{0,2} = F_{2,0} = 0\,, \qquad \Omega^{ij} F_{ij} = 0\,, \tag{4.1}$$

then it is straightforward to show that A solves the field equations

$$\nabla^i F_{ij} = 0\,. \tag{4.2}$$

The proof makes use of the Jacobi identities for F. Donaldson has shown that if E is a stable bundle over a complex surface M, then there is a unique connection which solves (4.1).

Next suppose that E is a vector bundle over a *non*-Kähler conformally-balanced KT manifold (M, J, G) equipped with a connection A with curvature F. Donaldson equations (4.1) can be easily generalised to KT manifolds by allowing Ω to be the Kähler form of the hermitean metric G. We shall show that the Donaldson equations imply the field equations

$$\nabla^{(+)i}(e^{-2\Phi} F(A)_{ij}) = 0\,, \tag{4.3}$$

where $\nabla^{(+)}$ is the connection of the KT structure (M, J, G) with torsion H and Lee form $\theta = 2d\Phi$. For this we choose complex coordinates with respect to the complex structure J and rewrite (4.3) as

$$-2\nabla_\gamma G^{\gamma\bar\beta} F_{\bar\beta\alpha} + G^{\gamma\bar\beta}\nabla_\gamma F_{\bar\beta\alpha} - \frac{1}{2} H^{\bar\delta}{}_{\gamma\bar\beta} F_{\bar\delta\alpha} G^{\gamma\bar\beta} - \frac{1}{2} H^{\delta}{}_{\gamma\alpha} F_{\bar\beta\delta} G^{\gamma\bar\beta} = 0\,.$$

The Jacobi identities imply that

$$\nabla_\gamma F_{\bar\beta\alpha} + \nabla_\alpha F_{\gamma\bar\beta} = -\nabla_{\bar\beta} F_{\alpha\gamma}\,,$$

where

$$\nabla_{\bar\beta} F_{\alpha\gamma} = \frac{1}{2} H^{\bar\delta}{}_{\bar\beta\alpha} F_{\bar\delta\gamma} + \frac{1}{2} H^{\bar\delta}{}_{\bar\beta\gamma} F_{\alpha\bar\delta}\,,$$

and ∇ is the Levi-Civita connection of the metric G. Collecting the various terms together, we find that

$$(\theta_\gamma - 2\partial_\gamma\Phi) G^{\gamma\bar\beta} F_{\bar\beta\alpha} = 0$$

which vanishes identically because (M, J, G) is conformally balanced, $\theta = 2d\Phi$. Therefore equations (4.1) together with the Jacobi equations imply the field equations (4.3).

4.2 The gauge field equations

We shall use the result of the previous section to show that the field equations of the gauge field are satisfied provided that the Killing-spinor equations (2.4) are satisfied.

Assuming that the background (g,T,φ,B) and its deformation (G,H,Φ,A) satisfy the Killing-spinor equations (2.4), the KT structures (X_n,J,g) and (X_n,J,G) are conformally balanced and the holonomies of their $\nabla^{(+)}$ connections are contained in $\mathrm{SU}(n)$. In addition both gauge connections B and its deformation A satisfy the conditions (4.1) — the latter up to linear order in α'.

Applying the theorem proven in the previous section, we conclude that the background (g,T,φ,B) and its deformation (G,H,Φ,A) satisfy the field equations (4.3) — the latter up to linear order in α'. Expanding (4.3) in α' for the background (G,H,Φ,A) and since (4.3) is satisfied at zeroth order in α', the linear term in α' will vanish identically. Thus if the background (g,T,φ,B) and its deformation (G,H,Φ,A) satisfy the Killing-spinor equations, then the field equations (2.3) for the gauge potential are satisfied without additional conditions on the deformations. The conditions on the deformation Q of the gauge connection B are

$$\begin{aligned}\nabla_\alpha Q_\beta-\nabla_\beta Q_\alpha=\nabla_{\bar\alpha}Q_{\bar\beta}-\nabla_{\bar\beta}Q_{\bar\alpha}&=0\\ h^{\alpha\bar\beta}F(B)_{\alpha\bar\beta}+g^{\alpha\bar\beta}\left(\nabla_\alpha Q_{\bar\beta}-\nabla_{\bar\beta}Q_\alpha\right)&=0\,,\end{aligned}\tag{4.4}$$

where the covariant derivative ∇ is with respect to the gauge connection B. We have derived these by substituting the deformation (G,H,Φ,A) of (g,T,φ,B) in the Killing-spinor equations (4.1) and by collecting the terms linear in α'.

It remains to find whether the Killing-spinor equations for the gaugino or equivalently (4.1) have solutions on a general conformally balanced KT manifold. It is easy to investigate the case where A is an abelian connection. However, the non-abelian case is more involved and so we shall not pursue this further.

5. (2,0) heterotic compactifications

The compactification ansätze for the heterotic string which preserve (2,0) world-volume supersymmetry and are spacetime supersymmetric, are given in (3.1) with the additional assumption that the internal space X_n is compact.[9] As we have discussed, such backgrounds are expected to receive α' corrections. Using the machinery developed in the previous sections, we shall investigate the deformations of these backgrounds due to α' corrections.

5.1 The zeroth-order solution

As we have explained the requirement for a compactification to preserve (2,0) world-sheet supersymmetry and 2^{1-n} of spacetime supersymmetry in $(10-2n)$-dimensions leads to an internal manifold X_n with a conformally balanced KT structure and $\mathrm{hol}(\nabla^{(+)})\subseteq\mathrm{SU}(n)$.

[9] The ansatz (3.1) was considered in [10] where it was shown that no warp factor is allowed for the non-compact part of the metric. Therefore, as is easy to see using sigma-model perturbation theory, there can be no such warp factor in (3.2) either.

The additional requirement that X_n is compact,[10] the implicit assumption that all the fields (g, T, φ) are smooth on X_n and the fact that at zeroth order in α' $dT = 0$, impose strong restrictions on the geometry of X_n. It has been shown in [15] under the above assumptions[11] that X_n *is* Calabi-Yau , $T = 0$ and the dilaton φ is constant. In addition at zeroth order in α' the gauge connection B satisfies the Donaldson equations (4.1) on the Calabi-Yau manifold X_n. These data are the starting point of our α' expansion.

The Calabi-Yau background (g, φ, B) receives α' corrections. The deformation (G, H, Φ, A) of (g, φ, B) has non-vanishing torsion H which is not closed, $dH \neq 0$, as required by the anomaly-cancellation mechanism. Note that the zeroth-order term in H vanishes and so H is purely first-order in α', $H = \alpha' f + O(\alpha'^2)$.

5.2 The first-order solution

To find the correction (G, H, Φ, A) to the Calabi-Yau geometry (g, φ, B), we have to solve equations (3.6) and (4.4) for the first-order deformations (h, f, ϕ, Q). The deformation ϕ to the dilaton is given in (3.9), $\phi = \frac{1}{4} g^{\beta\bar{\gamma}} h_{\beta\bar{\gamma}}$. Similarly, the deformation f of the torsion is given in (3.4). The remaining field and Killing-spinor equations are satisfied without further conditions.

Since for this background the torsion vanishes at zeroth order in α', condition (3.6) arising from the requirement that $\mathrm{hol}(\nabla^{(+)}) \subset \mathrm{SU}(n)$, can be rewritten as

$$\nabla^{\bar{\beta}} h_{\alpha\bar{\beta}} - \frac{1}{2} \nabla_\alpha \left(g^{\gamma\bar{\beta}} h_{\gamma\bar{\beta}} \right) = 0 \,. \tag{5.1}$$

The above equation can be thought of as a gauge-fixing condition for the deformations associated with infinitesimal diffeomorphisms of the underlying manifold. Condition (5.1) can always be attained. To see this, first write (5.1) in real coordinates

$$\nabla^j h_{ji} - \frac{1}{4} \nabla_i h^j{}_j = 0 \,. \tag{5.2}$$

Suppose that h does not solve (5.2). We shall show that there is a v such that h' given by $h'_{ij} = h_{ij} + \nabla_i v_j + \nabla_j v_i$ satisfies (5.2). v is determined by

$$\nabla^k \nabla_k v_i + \frac{1}{2} \nabla_i \nabla^k v_k = \nabla^j h_{ji} - \frac{1}{4} \nabla_i h^j{}_j \,. \tag{5.3}$$

First note that the Kernel of the operator on the left-hand-side of the equation above is zero on an *irreducible* Calabi-Yau manifold. Indeed let v be in the Kernel. Then

$$\int_{X_n} (v^i \nabla^k \nabla_k v_i + \frac{1}{2} v^i \nabla_i \nabla^k v_k) \; d\mathrm{vol} = - \int_M \left(\nabla_k v_i \nabla^k v^i + \frac{1}{2} (\nabla^k v_k)^2 \right) d\mathrm{vol} = 0 \,.$$

Thus v is parallel with respect to the Levi-Civita connection. Since X_n is irreducible, there are no parallel one-forms on X_n and so the Kernel vanishes. Therefore equation (5.3) can be solved for v since the right-hand-side does not vanish, by assumption.

[10] This assumption is sufficient for the spectrum in $(10 - 2n)$ dimensions to be discrete.

[11] In fact in [15] it was shown that X_n is Calabi-Yau even if $dT \neq 0$ provided a certain inequality holds.

We can also determine the metric moduli of (2,0) compactifications. Since the $\partial\bar{\partial}$-lemma applies for Calabi-Yau manifolds, the Aeppli group $V^{1,1}(X_n)$ is isomorphic to the the Hodge group $H^{1,1}(X_n)$. Thus the dimension of the moduli space is the Hodge number $h^{1,1}$ which is the same as the number of metric moduli of Calabi-Yau manifolds. Of course one can also take into account the moduli associated with including in the theory NS two-form gauge potentials. The latter are harmonic (1,1)-forms. This leads to a complex moduli space of real dimension $2h^{1,1}$. One concludes that the metric moduli of Calabi-Yau (2,0)-compactifications, which are associated with NS fluxes induced by the heterotic anomaly, are *not* lifted. An effect of the heterotic anomaly-cancellation mechanism is a shift in the origin of the moduli space.

The moduli of the theory associated with deformations of the complex structure of the underlying Calabi-Yau manifold is not lifted either. The analysis that we have done using the complex structure J can be repeated with any other complex structure on the Calabi-Yau manifold. Therefore, we conclude that the presence of NS flux associated with the heterotic anomaly cancellation mechanism does *not* lift the Calabi-Yau moduli at this order in α' perturbation theory.

6. The α'-corrected conifold

The conifold is a singular non-compact Calabi-Yau threefold [17]. Here we apply the machinery of the previous sections to compute the α' corrections explicitly.

The conifold can be thought of as a Ricci-flat cone over a $T^{1,1}$ space, where the latter is a particular U(1) fibration over $S^2 \times S^2$ [25]. Let $0 \le \phi_i \le 2\pi$, $0 \le \theta_i \le \pi$, $i = 1, 2$ be angular coordinates parametrising the two spheres S^2, $0 \le \psi \le 4\pi$ be the coordinate on the U(1) fibre and $\rho \ge 0$ be the radial coordinate. The line element of the conifold is

$$\begin{aligned} ds^2 = g_{mn} dx^m dx^n &= d\rho^2 + \frac{\rho^2}{9}\left(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2\right)^2 + \\ &\quad + \frac{\rho^2}{6}\left(\sin^2\theta_1 d\phi_1^2 + d\theta_1^2\right) + \frac{\rho^2}{6}\left(\sin^2\theta_2 d\phi_2^2 + d\theta_2^2\right) . \end{aligned} \tag{6.1}$$

The second term on the right-hand-side is the vertical displacement along the U(1) fibre whereas the last two terms represent the line element of $S^2 \times S^2$. There is a conical singularity at $\rho = 0$, where the $T^{1,1}$ base of the cone shrinks to zero size. It is useful to note that $T^{1,1}$ is topologically $S^2 \times S^3$.

Let us now consider a deformation $g_{mn} \to g_{mn} + \alpha' h_{mn}$, where h_{mn} is hermitean,

$$h_{kl} J^{\kappa}{}_m J^l{}_n = h_{mn} . \tag{6.2}$$

The complex structure J of the conifold is

$$\begin{aligned} J^{\theta_1}{}_{\phi_1} &= \sin\theta_1 , & J^{\rho}{}_{\phi_1} &= -\frac{\rho}{3}\cos\theta_1 , \\ J^{\theta_2}{}_{\phi_2} &= \sin\theta_2 , & J^{\rho}{}_{\phi_2} &= -\frac{\rho}{3}\cos\theta_2 , \\ J^{\phi_1}{}_{\theta_1} &= -\frac{1}{\sin\theta_1} , & J^{\psi}{}_{\theta_1} &= \cot\theta_1 , \\ J^{\phi_2}{}_{\theta_2} &= -\frac{1}{\sin\theta_2} , & J^{\psi}{}_{\theta_2} &= \cot\theta_2 , \\ J^{\rho}{}_{\psi} &= -\frac{\rho}{3} , & J^{\psi}{}_{\rho} &= \frac{3}{\rho} , \end{aligned} \tag{6.3}$$

and the rest of the components vanish. Condition (6.2) implies that h_{mn} is of the form,

$$\begin{aligned} h_{mn}dx^m dx^n &= D\left[d\rho^2 + \frac{\rho^2}{9}(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2\right] + \\ &\quad + A(\sin^2\theta_1 d\phi_1^2 + d\theta_1^2) + C(\sin^2\theta_2 d\phi_2^2 + d\theta_2^2) + \\ &\quad + 2B\sin\psi(\sin\theta_1 d\phi_1 d\theta_2 - \sin\theta_2 d\phi_2 d\theta_1) + \\ &\quad + 2B\cos\psi(d\theta_1 d\theta_2 + \sin\theta_1\sin\theta_2 d\phi_1 d\phi_2)\,. \end{aligned} \tag{6.4}$$

We take A, B, C, D to be functions of the radial coordinate alone. With this assumption, it can be seen from the form of most general $T^{1,1}$ metric given in [26], that (6.4) is a foliation of $T^{1,1}$ spaces.

The gauge-fixing condition (5.2) is equivalent to

$$\begin{aligned} 0 &= A + C + \frac{3}{2}\rho(A+C)' - \frac{1}{12}\rho^3 - \frac{2}{3}\rho^2 F \\ 0 &= B\,. \end{aligned} \tag{6.5}$$

In order to solve the Einstein equation, we need to find

$$S_{mn} := \frac{1}{4}R_m{}^{kls}R_{nkls}\,.$$

After some computation, we get

$$S_{\phi_1\phi_1} = S_{\phi_2\phi_2} = \frac{\sin^2\theta}{\rho^2}\,, \qquad S_{\theta_1\theta_1} = S_{\theta_2\theta_2} = \frac{1}{\rho^2}\,. \tag{6.6}$$

The Einstein equation (3.10) (or its gauged-fixed version (C.4) of appendix C) is then equivalent to the following set of conditions,

$$\begin{aligned} 0 &= B \\ 0 &= A + C + \frac{\rho^2}{24}(-8D + 5\rho D' + \rho^2 D'') \\ 0 &= 6\rho A' + 6\rho^2 A'' + \rho^2(-4D + 5\rho D' + \rho^2 D'') - 3 \\ 0 &= A' + \rho A'' - C' - \rho C''\,. \end{aligned} \tag{6.7}$$

Demanding that the total metric be asymptotic to the conifold in the region $\rho/\sqrt{\alpha'} \to \infty$, the general solution to order $\mathcal{O}(\alpha')$ reads

$$\begin{aligned} A &= -\frac{1}{16}\left(\frac{3}{2} - c_1 - c_2\,\log\frac{\rho}{\rho_0}\right) \\ B &= 0 \\ C &= -\frac{1}{16}\left(\frac{3}{2} + c_1 + c_2\,\log\frac{\rho}{\rho_0}\right) \\ D &= -\frac{3}{8\rho^2}\,, \end{aligned} \tag{6.8}$$

where ρ_0 is a dimensionful constant introduced to make the constants c_1, c_2 dimensionless. Note that the gauge-fixing condition (6.5) is automatically satisfied by the solution (6.8). Therefore the solution is spacetime supersymmetric.

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To summarise, the $\mathcal{O}(\alpha')$-perturbed metric is

$$\begin{aligned} ds^2 =& \left(1-\frac{3\alpha'}{8\rho^2}\right)\left[d\rho^2+\frac{\rho^2}{9}\left(d\psi+\cos\theta_1 d\phi_1+\cos\theta_2 d\phi_2\right)^2\right]+\\ &+\frac{\rho^2}{6}\left[1-\frac{3\alpha'}{8\rho^2}\left(\frac{3}{2}-c_1-c_2\ log\frac{\rho}{\rho_0}\right)\right]\left(\sin^2\theta_1 d\phi_1^2+d\theta_1^2\right)+\\ &+\frac{\rho^2}{6}\left[1-\frac{3\alpha'}{8\rho^2}\left(\frac{3}{2}+c_1+c_2\ \log\frac{\rho}{\rho_0}\right)\right]\left(\sin^2\theta_2 d\phi_2^2+d\theta_2^2\right). \end{aligned} \tag{6.9}$$

The perturbation is valid in the regime $\rho/\sqrt{\alpha'}\gg 1$. In order to be able to extract any information about the behavior of the metric near the apex ($\rho=0$), the full tower of α' corrections would have to be taken into account as these become important in the sub-stringy regime $\rho/\sqrt{\alpha'}\leq 1$. This is clearly beyond the validity of our analysis. Nevertheless, we can attempt to extrapolate our result to the region $\rho/\sqrt{\alpha'}\sim 1$. An analysis then reveals that at a radial distance of the order of $\sqrt{\alpha'}$, an S^2 in the $T^{1,1}$ base shrinks to zero volume. We therefore find that the singularity of the conifold persists, although it becomes milder in the sense that not the entire base shrinks to zero.

7. The α'-corrected $U(n)$-invariant Calabi-Yau metric

To describe the $U(n)$-invariant Calabi-Yau metric [18], we introduce complex coordinates $\{z^\alpha;\alpha=1,\ldots,n\}$ and consider the $U(n)$ invariant hermitean metric

$$ds^2=A(r^2)dz\cdot d\bar{z}+B(r^2)\bar{z}\cdot dz\ z\cdot d\bar{z}\,,$$

where $r^2=\delta_{\alpha\bar{\beta}}z^\alpha z^{\bar{\beta}}$, $dz\cdot d\bar{z}=\delta_{\alpha\bar{\beta}}dz^\alpha dz^{\bar{\beta}}$ and similarly for the rest. The metric is Kähler if

$$B=A'\,,$$

where the prime denotes differentiation with respect to r^2. The Levi-Civita connection one-form is

$$\Gamma^\alpha{}_\beta=A^{-1}A'(\delta^\alpha{}_\beta\bar{z}\cdot dz+\bar{z}_\beta dz^\alpha)+\frac{A''-2A^{-1}(A')^2}{A+r^2A'}z^\alpha\bar{z}_\beta\bar{z}\cdot dz\,.$$

The holonomy of the above connection is contained $U(n)$. The metric is Calabi-Yau if and only if the holonomy of the above connection is in $\mathrm{SU}(n)$. The connection of canonical bundle is

$$\omega_\alpha=i\partial_\alpha\ln\det(g_{\alpha\bar{\beta}})\,,$$

where $\det(g_{\alpha\bar{\beta}})=A^{n-1}(A+r^2A')$. The curvature of canonical bundle vanishes iff

$$A^n+\frac{r^2}{n}(A^n)'=\lambda\,,$$

where λ is a constant. The most general solution of this equation is

$$A^n=\lambda+\frac{c}{r^{2n}}\,,$$

where c is constant. This metric is the Calabi-Yau metric on the orbifold $\mathbb{C}^n/\mathbb{Z}_n$ after resolving the singularity at the origin by replacing with a $\mathbb{C}P^2$. Next we compute $P = \frac{1}{4}\mathrm{tr}R^2$ to find

$$P = \frac{1}{4}C(dz \wedge d\bar{z}) \wedge (dz \wedge d\bar{z}) + \frac{1}{4}C'\bar{z} \cdot dz \wedge z \cdot d\bar{z} \wedge (dz \wedge d\bar{z}) \,,$$

where

$$C = (n+1)A^{-2}(A')^2 + 2A^{-1}A'r^2\frac{A'' - 2A^{-1}(A')^2}{A + r^2A'} + r^4\left(\frac{A'' - 2A^{-1}(A')^2}{A + r^2A'}\right)^2$$

and $(dz \wedge d\bar{z}) = \delta_{\alpha\bar{\beta}}\; dz^\alpha \wedge dz^{\bar{\beta}}$. After some computation, we find that

$$C = n(n+1)\left(\frac{c}{\lambda r^{2n+2} + cr^2}\right)^2 .$$

In addition setting

$$h = D(r^2)dz \cdot d\bar{z} + E(r^2)\bar{z} \cdot dz\; z \cdot d\bar{z} \,,$$

and $Y_{\alpha\bar{\beta}} = -ih_{\alpha\bar{\beta}}$, we find that $P = -2i\partial\bar{\partial}Y$ implies

$$D' - E = -\frac{1}{8}C \,. \tag{7.1}$$

Condition (5.1) for h, gives

$$2(n-1)A^{-1}E - (n-1)A^{-1}D' - (n-1)A^{-2}A'D + \frac{1}{\lambda}[A^{n-1}(D + r^2E)]' = 0 \,. \tag{7.2}$$

The supersymmetric deformation can be computed by solving (7.1) and (7.2). Substituting for E in (7.2), we find after some algebra that

$$\begin{aligned} 0 = &- \frac{c^2n(n+1)}{8\lambda r^{4n+4}}\frac{3\lambda + \frac{nc}{r^{2n}}}{(\lambda + \frac{c}{r^{2n}})^2} - \frac{c^2(n-1)}{\lambda r^{4n+2}}\frac{1}{(\lambda + \frac{c}{r^{2n}})}D + \\ &+ \left[\frac{n-1}{\lambda}\left(\lambda - \frac{c}{r^{2n}}\right) + 2\left(\lambda + \frac{c}{r^{2n}}\right)\right]D' + \frac{r^2}{\lambda}\left(\lambda + \frac{c}{r^{2n}}\right)D'' \,. \end{aligned} \tag{7.3}$$

We have not been able to obtain a closed form for D. But we can solve the equation perturbatively in a large distance expansion. The result reads,

$$\begin{aligned} E &= \frac{n(c/\lambda)^2}{8r^{4n+4}}\left(n - 2 + \frac{-5n^2 + 8n + 1}{2n+1}\frac{(c/\lambda)}{r^{2n}} + \cdots\right) \\ D &= \frac{n(c/\lambda)^2}{8r^{4n+2}}\left(\frac{3}{2n+1} + \frac{n^2 - 14n - 3}{(2n+1)(3n+1)}\frac{(c/\lambda)}{r^{2n}} + \cdots\right) . \end{aligned} \tag{7.4}$$

Note that the leading correction to the metric behaves like r^{-4n-2}.

8. Concluding remarks

Compactifications with fluxes lead to lower-dimensional effective theories which exhibit potentials lifting some of the moduli. The relevant flux for (2,0) heterotic-string compactifications is the NS three-form H. There are two possibilities. One is to allow for a non-vanishing flux at zeroth order in α'. Then the internal manifold is either non-compact or/and some of the fields are singular. We have extensively investigated the case of Calabi-Yau compactifications where the NS flux vanishes at zeroth order in α'. As we have shown such compactifications develop a non-vanishing flux H and a non-constant dilaton at first order in α'. In addition, at this order in α', the compactifications have the same dimension of moduli space as that of the moduli space of the Calabi-Yau manifolds at zeroth order in α' and so there is no lifting of moduli.

The effective lower-dimensional theories which arise in standard Calabi-Yau compactifications, i.e. without NS flux, may be different from those with flux. If there is no NS flux induced by the heterotic anomaly-cancellation mechanism, then it is consistent to take the lower-dimensional effective action to be the standard ten-dimensional N=1 supergravity action. Of course there will be higher-order α' corrections. Nevertheless it is consistent to consider only the zeroth order in α'. On the other hand, if one wishes to consider the NS fluxes induced by the heterotic anomaly-cancellation mechanism, *consistency requires* that curvature square terms in the field equations and their supersymmetric completion should be considered as well. These will contribute to the lower-dimensional effective theories.

The other possibility is to introduce a non-vanishing H flux at zeroth order in α' and to allow for some fields to be singular or/and for the internal manifold to be non-compact. In the case that the manifold or the fields are singular, one may expect that these singularities can be resolved by taking into account α' corrections. We have seen, for example, that after taking into account the linear order α' correction to the conifold, the metric is less singular but the singularity is not completely resolved. In a scenario where the singularities are removed and the internal manifold remains compact, some moduli may be lifted and potentials of the type given in [28] may be generated in the effective theory in lower dimensions.

Similar conclusions can be drawn for compactifications of other theories with fluxes in the presence of an anomaly-cancellation mechanism. Such an example is M-theory [27]. To investigate the consistency of the anomaly cancellation with spacetime supersymmetry as we have done for the heterotic string, one needs to know the higher-order derivative corrections to $D = 11$ supergravity and their supersymmetric completions.

It is of interest to speculate on the geometry of the background (3.1) after all α' corrections are taken into account. It is expected that such a background is a hermitean manifold, based on the assumption that (2,0) world-sheet supersymmetry is preserved in sigma-model perturbation theory. It is unlikely that the supergravity Killing-spinor equations will remain of the form (2.4). The dependence on the metric and the torsion will change and higher-curvature terms are expected to appear [23]. So the conditions for preserving spacetime supersymmetry will not be simply expressed as conditions on the holonomy of the $\nabla^{(+)}$ connection and on the Lee form θ of the hermitean geometry. However, some properties of the underlying manifold may be preserved. It is plausible to assume that the manifold satisfies the $\partial\bar{\partial}$-lemma and the canonical bundle is holomorphically trivial. It has been shown in [24] that Moishezon manifolds with these properties admit a connection with skew-symmetric torsion and holonomy contained in SU(n). However, it is not apparent that the associated metric is that which arises after taking into account all α' corrections. In addition, on such non-Kähler Moishezon manifolds there is no KT structure for which the associated three-form field strength is closed [24]. Therefore such manifolds cannot be used as the starting point of the sigma-model perturbation theory.

Acknowledgments

We would like to thank P. Howe for stimulating discussions. This work was partially supported by PPARC grants PPA/G/S/1998/00613 and PPA/G/O/2000/00451 and by EU grant HPRN-2000-00122.

A. Useful formulae for KT geometry

Let (X_n, J, G) be a KT manifold, i.e. X_n is a hermitean manifold of complex dimension n with metric G and complex structure J such that $\nabla^{(+)}J = 0$, where $\nabla^{(+)}$ has skew-symmetric torsion H. In the mathematics literature $\nabla^{(+)}$ is called the Bismut connection. The holonomy of $\nabla^{(+)}$ is contained in $U(n)$. In complex coordinates, the holonomy condition requires $\Gamma^{(+)\alpha}_{i}{}_{\bar{\beta}} = 0$ which in turn gives

$$H_{\alpha\beta\bar{\gamma}} = -\partial_\alpha G_{\beta\bar{\gamma}} + \partial_\beta G_{\alpha\bar{\gamma}} \,. \tag{A.1}$$

The rest of the components of the torsion are determined by complex conjugation. The (3,0) and (0,3) components of H vanish as it can be seen from the integrability of the complex structure. So H is determined uniquely in terms of the metric and complex structure of (X_n, J, G). The Lee form of the KT geometry is

$$\theta_i = \frac{1}{2} J^j{}_i H_{jkl} \Omega^{kl} \,,$$

where $\Omega_{ij} = G_{ik} J^k{}_j$ is the Kähler form. In complex coordinates, the Lee form can be written as

$$\theta_\alpha = \partial_\alpha \ln\det(G_{\beta\bar{\gamma}}) - G^{\beta\bar{\gamma}} \partial_\beta G_{\alpha\bar{\gamma}} \tag{A.2}$$

and $\theta_{\bar{\alpha}} = (\theta_\alpha)^*$.

The connection of the canonical bundle induced by $\nabla^{(+)}$ is

$$\omega_\alpha = i\Gamma^{(+)}{}_\alpha{}^\beta{}_\beta - i\Gamma^{(+)}{}_\alpha{}^{\bar{\beta}}{}_{\bar{\beta}} = 2iG^{\beta\bar{\gamma}}\partial_\beta G_{\alpha\bar{\gamma}} - iG^{\beta\bar{\gamma}}\partial_\alpha G_{\beta\bar{\gamma}} \tag{A.3}$$

and $\omega_{\bar{\alpha}} = (\omega_\alpha)^*$. Let $\rho = d\omega$ be the curvature of the U(1) connection ω. The holonomy of the connection $\nabla^{(+)}$ is contained in SU(n), iff $\rho = 0$.

A KT manifold is conformally balanced iff there is a function Φ on X_n such that $\theta = 2d\Phi$, i.e. the Lee form is exact. It has been shown in [16] that if (X_n, J, G) is a conformally balanced KT manifold and the holonomy of $\nabla^{(+)}$ is contained in $\mathrm{SU}(n)$ $(\rho = 0)$, then

$$R_{ij} + \frac{1}{4} H^k{}_{il} H^l{}_{jk} + 2\nabla_i \partial_j \Phi = \frac{1}{4} J^k{}_i (dH)_{kjmn} \Omega^{mn} \,, \tag{A.4}$$

where ∇ is the Levi-Civita connection of the metric G. Note that we do *not* require $dH = 0$ in the above expression.

Our conventions for the curvature of a connection Γ are

$$R_{ij}{}^k{}_l = \partial_i \Gamma^k_{jl} - \partial_j \Gamma^k_{il} + \Gamma^k_{im} \Gamma^m_{jl} - \Gamma^k_{jm} \Gamma^m_{il}$$

and the Ricci tensor is $R_{ij} = R_{mi}{}^m{}_j$.

B. Lichnerowicz and Laplace operators

Let (M, g) be a riemannian manifold with associated Levi-Civita connection ∇. The Lichnerowicz operator Δ_L is defined by

$$R_{ij}(g + \epsilon h) = R_{ij} + \epsilon \Delta_L h_{ij} + O(\epsilon^2) \,.$$

So Δ_L is the first-order deformation of the Ricci tensor. A calculation reveals that

$$\begin{aligned} \Delta_L h_{ij} = & -\frac{1}{2} \nabla^2 h_{ij} - R_{ikjl} h^{kl} + \frac{1}{2} \nabla_i \nabla^k h_{kj} + \frac{1}{2} \nabla_j \nabla^k h_{ki} - \\ & -\frac{1}{2} \nabla_i \nabla_j h^k{}_k + \frac{1}{2} R_{ki} h^k{}_j + \frac{1}{2} R_{kj} h^k{}_i \,. \end{aligned} \tag{B.1}$$

The laplacian operator Δ on a two form Y is

$$\Delta Y_{ij} = -\frac{1}{2} \nabla^k \nabla_k Y_{ij} - R_{ikj\ell} Y^{k\ell} + \frac{1}{2} R_{ik} Y^k{}_j - \frac{1}{2} R_{jk} Y^k{}_i \,.$$

On Ricci-flat Kähler manifolds, we can relate the Lichnerowicz operator to the Laplace operator on two-forms by choosing the gauge fixing condition

$$\nabla^j h_{ji} - \frac{1}{2} \nabla_i h^j{}_j = 0$$

for infinitesimal diffeomorphisms and by using the relation $Y_{ij} = h_{ik} J^k{}_i$ between symmetric (1,1)-tensors and (1,1)-forms. The hermiticity condition for the metric implies that the deformations h of the metric are (1,1) tensors.

Note that the above gauge can always be attained. Suppose that h does not satisfy the gauge. Assume that there is an infinitesimal diffeomorphism v such that $h'_{ij} = h_{ij} + \nabla_i v_j + \nabla_j v_i$ satisfies the gauge. Then v is determined by the equation

$$\nabla^k \nabla_k v_i = \nabla^j h_{ji} - \frac{1}{2} \nabla_i h^j{}_j \,.$$

To show that there is always such a v, we must invert the above equation. This can be achieved iff $\nabla^j h_{ji} - \frac{1}{2} \nabla_i h^k{}_k$ is orthogonal to the kernel of the elliptic operator $\nabla^k \nabla_k$. We rewrite the above equation as

$$(d\delta + \delta d) V = -\nabla^j Y_{ji} + \frac{1}{2} J^k{}_i \nabla_k h^j{}_j \,,$$

where $V_i = v_k J^k{}_i$ and δ is the adjoint of d. Of course this equation can be inverted iff the right-hand-side is orthogonal to harmonic one-forms. Indeed let Z be a harmonic one-form, then

$$\int_M Z^i \left(-\nabla^j Y_{ji} + \frac{1}{2}\Omega_{ki}\nabla^k h^j{}_j \right) d\text{vol} = -\int_M \left(\nabla_i Z_j Y^{ij} + \frac{1}{2}\nabla_i Z_j \ \Omega^{ij} \ h^k{}_k \right) d\text{vol} = 0$$

because Z is closed.

C. Another derivation of the field equations for (2,0) compactifications

The first-order corrections (h, f, ϕ) to the background of a Calabi-Yau compactification (g, φ), $T = 0$, can be determined by the field equations without using the conditions for spacetime supersymmetry. To show this, we substitute (2.5) in the field equation (2.3) and collect the linear terms in α'. Using the fact that at zeroth order in α' the geometry is Calabi-Yau, the equation for the metric gives

$$\Delta_L h_{ij} + S_{ij} + 2\nabla_i \partial_j \phi = 0 \,, \tag{C.1}$$

where Δ_L is the Lichnerowicz operator (see appendix B). In this case, we have

$$\Delta_L h_{ij} = -\frac{1}{2}\nabla^2 h_{ij} - R_{ikjl}h^{kl} + \frac{1}{2}\nabla_i\nabla^k h_{kj} + \frac{1}{2}\nabla_j\nabla^k h_{ki} - \frac{1}{2}\nabla_i\nabla_j h^k{}_k \tag{C.2}$$

because X_n is Calabi-Yau and $R_{ij} = 0$, and

$$S_{ij} = \frac{1}{4}\left[R_{iklm}R_j{}^{klm} - F_{ikab}F_j{}^{kab} \right] . \tag{C.3}$$

The covariant derivative ∇ in (C.1) and (C.2) and the curvature R are with respect to the Calabi-Yau metric g.

To solve equation (C.1) with respect to h, we shall exploit the well-known relation between the Lichnerowicz and Laplace operators on Calabi-Yau manifolds. There are two ways to do this. One is to use the scheme dependence of the two-loop beta function and the other is to impose a gauge fixing condition on the deformations h of the metric g. The latter is common in moduli problems in order for the deformation h to be orthogonal to the orbits of infinitesimal diffeomorphisms. Using the scheme dependence of the two-loop beta function, we can arrange so that the term

$$\frac{1}{2}\nabla_i\nabla^k h_{kj} + \frac{1}{2}\nabla_j\nabla^k h_{ki} + \frac{1}{2} r \nabla_i \partial_j h^k{}_k$$

is cancelled by a wave function renormalization, where r is a real number. Alternatively, one can impose the gauge fixing condition

$$\nabla^k h_{ki} + \frac{r}{2}\nabla_i h^k{}_k = 0 \,.$$

Provided that we set for the deformation of the dilaton

$$\phi = \frac{1}{4}(r+1) h^k{}_k \,,$$

the remaining equation is

$$-\frac{1}{2}\nabla^2 h_{ij} - R_{ikjl}h^{kl} + S_{ij} = 0\,. \tag{C.4}$$

To solve this equation observe that S is a (1,1) symmetric tensor with respect to the complex structure. This allows us to consider deformations of the metric which are (1,1) as well, i.e. to take h to be a (1,1) symmetric tensor. It is well known that on Kähler manifolds there is a 1-1 correspondence between symmetric (1,1) tensors and (1,1) two-forms. Indeed define the forms $Y_{ij} = h_{ik}J^k{}_j$ and $Z_{ij} = S_{ik}J^k{}_j$ associated to T and S. Equation (C.4) becomes

$$\Delta Y + Z = 0\,, \tag{C.5}$$

where

$$\Delta Y_{ij} = -\frac{1}{2}\nabla^k\nabla_k Y_{ij} - R_{ikj\ell}Y^{k\ell}$$

is the standard Laplace operator on Y (see appendix B). We have taken into account that Calabi-Yau manifolds are Ricci-flat, $R_{ij} = 0$. So it remains to invert the Laplace operator to determine Y in terms of Z.

To solve the equation (C.5) in terms of Y, we have to show that Z is orthogonal to the harmonic two-forms of X_n in the Hodge decomposition with respect to the Calabi-Yau metric g. This can be achieved by relating the two-loop contribution Z to the beta function of the metric, to the heterotic anomaly P. Using

$$\Omega^{ij}R_{ijkl} = \Omega^{ij}F_{ijab} = 0\,,$$

we can show that

$$Z_{ij} = \frac{1}{4}P_{ijmn}\Omega^{mn}\,.$$

Now suppose that $n = 3$. After some computation, we find that

$$Z_{ij} = -\frac{1}{2}{}^*P_{ij} + \frac{1}{16}\Omega_{ij}P_{mnpq}\Omega^{mn}\Omega^{pq}\,.$$

The cancellation of the global anomaly implies that P is exact. As a consequence the dual two-form ${}^*P$ is co-exact and so it is orthogonal to harmonic two-forms. It remains to show that $P_{mnpq}\Omega^{mn}\Omega^{pq}$ is not harmonic. Observe that a harmonic function on X_3 is a non-vanishing constant. Integrating the identity

$$P \wedge \Omega = \frac{1}{8}P_{mnpq}\Omega^{mn}\Omega^{pq}d\mathrm{vol}$$

over the compact manifold X_3 and using that P is exact, it is easy to see that $P_{mnpq}\Omega^{mn}\Omega^{pq}$ is not harmonic.

This result can be easily extended to four- and eight-dimensional internal manifolds. The $n = 2$ case has been investigated in [8]. For $n = 4$, the computation is similar to $n = 3$. The only difference is the relation between the two-loop counterterm and the anomaly.

Next we investigate the field equation of the three-form field strength. Again substituting (2.5) in (2.3) and collecting the terms linear in α', we find

$$\nabla^i f_{ijk} = 0$$

or equivalently

$$d^\dagger f = 0\,,$$

where $d^\dagger$ is the adjoint operator of d. To derive this, we have used the fact that H vanishes at zeroth order in α'. In addition, we have that $df = -P$; f is well-defined because P is exact. Using the Hodge decomposition, we can write

$$f = f_h + dX + d^\dagger W\,,$$

where f_h is harmonic and W is a four-form. Adding the counterterm

$$b = -\alpha' X\,,$$

the exact three-form dX can be eliminated as follows

$$H = -\alpha' dX + \alpha' f = \alpha' f_h + \alpha' d^\dagger W\,.$$

Thus the field equation is satisfied and $dH = -P$. Alternatively, one can choose f such that $f = f_h + d^\dagger W$.

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